RN-6121/800

RN-6121

B. E. II (Sem. III) (EC/EL/CHEM) Examination

May / June – 2010

Engg. Maths - III

Time : **3** Hours] [Total Marks : **100**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **B. E. 2 (Sem. 3) (EC/EL/CHEM)**

Name of the Subject :
☛ **Engg. Maths - 3**

☛ Subject Code No. : **6 1 2 1** ☛ Section No. (1, 2,.....) : **1&2**

Seat No. :

Student's Signature

(2) Attempt **all** questions.
(3) Answer **each** section **separately**.
(4) Figures on **right** indicate marks.

SECTION - I

1 (a) Do as directed : **10**

(1) Express the integral $\int_0^\infty \int_0^\infty e^{-\left(x^2+y^2\right)} dx\, dy$ in the polar form.

(2) Find $div\,\overline{r}$ and $curl\,\overline{r}$ where $\overline{r}$ is position vector of $P(x, y, z)$.

(3) Show that $Curl\,(grad\ f) = 0$. Where f is a scalar point function.

(4) Define a solenoidal vector. Show that $\overline{F} = yz\,\hat{i} + zx\,\hat{j} + zx\,\hat{k}$ is solenoidal.

(5) Write the conditions, to be satisfied by a function, to have its Fourier series expansion.

(b) Attempt any **three** : **12**

(1) Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \dfrac{dy\, dx}{1+x^2+y^2}$.

(2) Find, by double integration, the area lying inside the circle $r = a\sin\theta$ and outside the cardioid $r = a(1-\cos\theta)$.

(3) Find the volume of the solid, bounded by the planes x=0, y=0, z=0 and x+y+z=1.

(4) Find the mass of a plane plate, in the form of one loop of lemniscate $r^2 = a^2 \cos 2\theta$ if the density at any point varies as square of its distance from the pole.

2 (a) Attempt any **two** : **16**

(1) Show that $\bar{F} = 2xyz\,\hat{i} + \left(x^2 z + 2y\right)\hat{j} + x^2 y\,\hat{k}$ is irrotational. Find its scalar potential function also.

(2) Prove that $div\left(grad\ r^n\right) = n(n+1)r^{n-2}$.

(3) Find the work done in moving a particle in the force field $\bar{F} = 3x^2\,\hat{i} + (2xz - y)\,\hat{j} + z\,\hat{k}$ along the straight line from (0,0,0,) to (2,1,3).

(b) Attempt any **two** : **8**

(1) Apply Green's theorem to prove that the area enclosed by a plane curve C is $\frac{1}{2}\int_C (x\,dy - y\,dx)$.
Hence find the area of an ellipse whose semi major and minor axes are of lengths a and b.

(2) If S is any closed surface enclosing a volume V and $\bar{F} = x\,\hat{i} + 2y\,\hat{j} + 3z\,\hat{k}$, prove that $\iint_S \bar{F}\cdot\hat{n}\,ds = 6V$.

(3) Evaluate by Stoke's theorem, $\oint_C (yz\,dx + zx\,dy + xy\,dz)$ where C is the curve $x^2 + y^2 = 1,\ z = y^2$.

3 (a) Find the Fourier sine series for $f(x) = \pi x - x^2$ in $(0, \pi)$. **4**

(b) Attempt any **two** : **10**

(1) Obtain Fourier series to represent $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in the interval $0 < x < 2\pi$.

(2) Obtain Fourier series for $f(x) = x^2$ in the interval $[-\pi, \pi]$. Hence deduce that $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$.

(3) Expand $f(x) = e^{-x}$ as a Fourier series in the interval $(-l, l)$.

SECTION - II

4 (a) Do as directed : **10**

(1) Define Gamma function. Derive $\Gamma(n+1) = n\,\Gamma(n), n > 0$.

(2) Express $\int_0^{\pi/2} \sin^{1/2}\theta \cos^{-1/2}\theta \, d\theta$ in terms of Beta function.

(3) Write one-dimensional wave equation. Write its physically acceptable solution also.

(4) Write Cauchy-Riemann equations in polar form.

(5) Define critical points of function $w = f(z)$. What are the critical points of function $f(z) = \cos z$?

(b) Attempt any **two** : **6**

(1) Show that $\int_0^1 \left(\log \frac{1}{x}\right)^{n-1} dx = \Gamma(n),\ n > 0$.

(2) Show that $\int_0^1 x^5 \left(1 - x^3\right) dx = \frac{1}{60}$.

(3) Show that $\int_a^b e^{-u^2} du = \frac{\sqrt{\pi}}{2}\left[erf(b) - erf(a)\right]$.

(c) Solve any **two** : **6**

(1) $x\left(y^2 - z^2\right)p + y\left(z^2 - x^2\right)q = z\left(x^2 - y^2\right)$

(2) $(mz - ny)\,p + (nx - lz)\,q = ly - mx$

(3) $pz - qz = z^2 + (x + y)^2$

5 Attempt any **two** : **12**

(1) An insulated rod of length l has its ends A and B maintained at 0°C and 100°C respectively until steady state conditions prevail. If B is suddenly reduced to 0°C and maintained at 0°C, find the temperature at a distance x from A at time t.

(2) A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $y = a \sin \frac{\pi x}{l}$ from which it is released at time t=0. Show that the displacement of any point at a distance x from one end at time t, is given by $y(x, t) = a \sin \frac{\pi x}{l} \cos \frac{\pi c t}{l}$.

(3) A rectangular plate with insulated surface is 8 cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature along one short edge y=0 is given by $u(x,0)=100\sin\frac{\pi x}{8},\ 0<x<8$ while the two long edges x=0 and x=8 as well as the other short edge are kept at 0°C, show that the steady state temperature at any point of the plate is given by

$$u(x,y)=100\,e^{-\pi y/8}\sin\frac{\pi x}{8},\ 0\le x\le 8.$$

6 (a) If f(z) is an analytic function of z, prove that **4**

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2=4|f'(z)|^2.$$

(b) Attempt any **two** : **6**

(1) Determine the analytic function whose real part is $\log\left(\sqrt{x^2+y^2}\right)$.

(2) Determine the region of the w-plane into which the first quadrant of z-plane is mapped by the transformation $w=z^2$.

(3) Find the image of the circle $|z-3|=5$ under the transformation $w=\frac{1}{z}$.

(c) Attempt any **two** : **6**

(1) Evaluate $\oint_C \frac{\cos\pi z}{z^2-1}dz$ where C is the circle $|z|=2$.

(2) Evaluate $\oint_C \frac{e^z}{(z+1)^2}dz$ where C' is the circle $|z|=3/2$.

(3) Evaluate $\oint_C \frac{2z+1}{z^2+z}dz$ where C is the circle $|z|=1/2$.